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Determining combined impact of psychological fear and media awareness in a classical SEIR model

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Abstract Classical $SEIR$ epidemic model comprised of four compartments - susceptible S, exposed E, infected I, and recovered R is a fundamental mechanism of epidemiological modeling that is enormously used in exploring vigorous dynamical traits of infectious diseases. To analyze the intricate dynamics of infectious diseases having noteworthy latent period, the utilization of SEIR models (consisting of exposed compartment E) is very beneficial. Onset of an epidemic outbreak in a certain geographical region triggers psychological fear in the community and this anxiety engenders relevant behavioral changes in human population during the epidemic progression. These behavioral changes along with awareness campaign coordinated by mass media (like Radio, TV, Newspaper, social media, YouTube etc) are capable of controlling the transmission of any communicable disease. However, some fraud news and rumors spread in mass media escalate the fear exponentially. In this article, the influence of psychological fear and the control of its consequential behavioral changes with the help of accurate mass media campaign are calibrated through a classical $SEIR$ mathematical model. The equilibrium points of the system and the stability conditions of the epidemic system around the equilibrium points are investigated. Sensitivity analysis is performed to measure the robustness of the model parameters in disease progression. Comprehensive numerical simulations are designed to portray different scenarios of the disease dynamics. The theoretical analysis and numerical simulations suggest that accurate mass media awareness is beneficial to control and diminish the psychological fear associated to an ongoing epidemic progression and its future outbreaks.

Keywords:

SEIR model, psychological fear, media awareness, sensitivity, stability

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Introduction:

1 Introduction:

Communicable diseases cast down substantial impacts on socio-economic structure of human population since very ancient times. The upshots of an emergent epidemic are not only limited to extensive deaths and morbidity, also causes extravagant deterioration in economy, social life, public health sector, education sector etc. worldwide. The human population in a certain epidemic region feels intense psychological fear that involves fear of losing life and sudden economic loss after emergence of any communicable disease. Psychological fear developed in human beings during an epidemic situation activates several behavioral changes that would curtail the chain of transmission, more precisely these behavioral changes are able to control the

effective contacts between susceptible humans and infected humans for a short period. Frightened human beings employ different intervention strategies to isolate themselves from social contacts [1]. Disease propagation is hampered due to self-imposed fear and thus it is advantageous in fragmented control of epidemic [3]. At the early stages of infection, government and public health sectors have no precise ideas regarding pharmaceutical interventions (medications, vaccination etc.) [2]. During emergence of an epidemic suddenly in a community, people started to be frightened regarding availability of proper medications and vaccination as well as side effects of these interventions [12]. At beginning stage of an infection, government and public health sectors focus on imposing non-pharmaceutical interventions like social distancing, wearing masks, frequent hand washing etc. along with work from home, online study, restrictions on traveling etc. [2, 4]. Appropriate information regarding non-pharmaceutical interventions is disseminated in different media like Radio, TV, Newspaper, social media, YouTube etc. that would be very helpful in lowering the level of infection [5-7]. However, counterfeit information regarding the etiological agents of any infection and several interventions strategies (viz. medications, vaccinations) conveyed by some fraud media (specially some fraud information spread in social media) proliferate rapidly like fire. At that instant, people in a certain community be more frightened resulting depletion in immunity. Long lasting level of anxiety or another form of psychological fear causes immune loss and frightened people would be more vulnerable to any infection. Authentic awareness via several health camps and

media is necessary to control self -imposed psychological fear of being infected as well as to inform individuals about available vaccination and its tenable side effects [6, 8, 9].

Epidemic models are being considered as one of the crucial tools to understand the vital dynamics of any communicable disease and these models also help to perceive preventive as well as control measures. A handful of mathematical models exist that established the role of mass media in disease dynamics [5, 6, 9–11]. These studies have explored that the effect of mass media awareness is one of the efficient and cost-effective measures in controlling infection. However, the impact of psychological fear in disease dynamics has not been explored in a large scale yet. In the study of Ghosh et al. [1], the authors have studied the impact of media and self-imposed psychological fear through an SI-type mathematical model of HIV/AIDS transmission dynamics. In, [9] the authors analyzed that if degree of fear is high, it will reduce the necessity of increase in the growth of media awareness taking the number of media advertisements as a dynamic variable. Yousef at al. [12], investigated the impact of fear triggered by different uncertain information spread in media on a community during COVID-19 pandemic situation. Being motivated from these studies, in our present study, we establish a classic SEIR compartmental model depicting the combined impact of accurate mass media awareness in controlling any communicable disease by curtailing the self-imposed psychological fear from a community.

Our present article is synchronized as follows: in Section 2, a classical SEIR ODE compartmental, deterministic mathematical model is formulated to capture the influence of accurate mass media awareness in controlling load of infection and psychological fear raised in a community. Section 3 is dealing with positivity and boundedness of the solutions of the system. In Section 4, the equilibrium points and the basic reproduction number of the system are computed. Section 5 is dealing with local dynamics of the epidemic system. Section 6 is dealing with global dynamics of the system. In Section 7 and Section 8, sensitivity analysis is performed to measure influence of the model parameters on overall disease dynamics of the endemic equilibrium and basic reproduction number respectively. In Section 9, various numerical simulations are attached to validate the analytical results biologically. Section 10 is composed of conclusions in respect of overall study

2 The Model formulation

Psychological fear (or, panic) is positively correlated with disease comorbidities, fast spreading of an infection, chronicity of the infection, disease induced deaths and socio-economic distress in a certain geographical region. Control of this kind of psychological fear is possible via accurate mass media awareness and pathological treatments. We construct an ODE-compartmental four dimensional classic $SEIR$ deterministic model calibrating the control of psychological fear of being infected and about side effects of available vaccine. In our proposed model, $S(t)$ is representing the class of susceptible individuals, $E(t)$ is representing the class of exposed individuals, $I(t)$ is indicating the class of infected individuals, and $R(t)$ is representing the class of recovered individuals at any instant t (measured in days). Thus, the coupled system of ordinary differential equations is as follows:

$$\begin{aligned} \frac{dS}{dt} &= \Lambda - (1-\rho)\frac{\beta SI}{1+\epsilon I} - \delta S, \\ \frac{dE}{dt} &= (1-\rho)\frac{\beta SI}{1+\epsilon I} - \gamma E - \delta E, \\ \frac{dI}{dt} &= \gamma E - \eta I - dI - \xi I - \delta I, \\ \frac{dR}{dt} &= \eta I + \xi I - \delta R, \end{aligned} \tag{1.1}$$

with non-negative biologically meaningful initial conditions needed for the dynamical study

$$S(0) > 0, E(0) \geq 0, I(0) \geq 0, R(0) \geq 0. \tag{2}$$

Here, the constant Λ is representing the constant recruitment of susceptible individuals in the epidemic system (1). The term β stands for the effective contact rate between susceptible individuals and infected individuals. The term ϵ stands for psychological fear which is able to generate saturation in the transmission of any infection. Here, ρ is denoting the accurate media awareness which is crucial in controlling the load of infection. δ is describing the natural death rate of each individual belong to the four compartments. At the rate γ, clinical symptoms appear and at the rate η, infected individuals are getting recovery naturally. Individuals being recovered at the rate ξ through pharmaceutical interventions. The term d stands for disease-induced death rate. All the parameters are positive and their parametric value for numerical simulation are enlisted in Table 1. The dynamical characteristics of the epidemic system (1) is showed in the Figure 1.

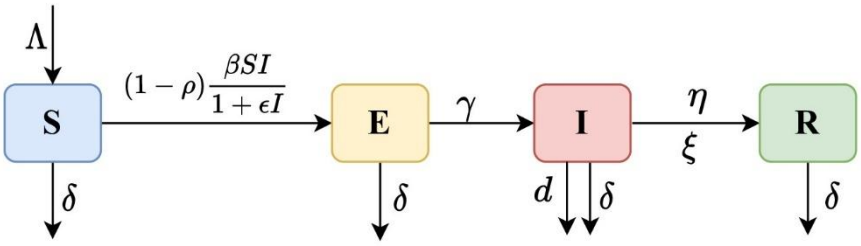

Figure 1: Schematic diagram of the epidemic system (1) portraying the flow of infection.

3 Basic characteristics of the model

In this section, the basic characteristics of the epidemic system (1) viz. positivity and boundedness of the system are investigated since the population could not be unbounded and negative any time.

3.1 Non-negativity of the solutions

Theorem 1. *All the solutions of the epidemic system (1) along with the initial values (2) are positively invariant in the interior of* R_{+}^{4}.

Proof. Rewriting the last equation of the epidemic system (1), we get

$$\frac{dR}{dt} \geq -\delta R.$$

Thus,

$$R(t) \geq R(0)\exp\left(-\int_0^t \delta\, dx\right) > 0, for\ all\ instant\ t\ > 0. \qquad (3)$$

Writing the third equation of the epidemic system (1), we get

$$\frac{dI}{dt} \geq -(\eta + d + \xi + \delta)I,$$

which implies that

$$I(t) \geq I(0)\exp\left(-\int_0^t (\eta + d + \xi + \delta)\, dx\right) > 0, for\ all\ instant\ t > 0. \qquad (4)$$

The second equation of the system (1) could be rewritten as

$$\frac{dE}{dt} \geq -(\gamma + \delta)E.$$

Integrating the above inequality, we obtain

$$E(t) \geq E(0)\exp\left(-\int_0^t (\gamma + \delta)\, dx\right) > 0, for\ all\ instant\ t > 0. \qquad (5)$$

Similarly, the first equation of the system equations (1) could be expressed as

$$\frac{dS}{dt} \geq -(1-\rho)\frac{\beta SI}{1+\epsilon I} - \delta S.$$

Integrating the above inequality, we get

$$S(t) \geq S(0)exp\left(-\int_0^t ((1-\rho)\frac{\beta S I}{1+\epsilon I} + \delta)\right) dx > 0 \text{ for all instant } t > 0. \qquad (6)$$

Therefore, all the solution trajectories $(S(t), E(t), I(t), R(t))$ of the system (1) together with the non-negative initial conditions (2) would be positive, for all instant t.

3.2 Boundedness

In this subsection, we investigate the boundedness of the solutions of the epidemic system (1), as the solutions should be well-posed.

Theorem 2. *All the solutions of the epidemic system (1) which initiate in* $\mathbb{R}_+^4$ *are uniformly bounded in the region* Γ *(defined in the proof), for all time* $t > 0$.

Proof. Summing up all the equations of the $SEIR$ system (1), we have

$$\frac{dP}{dt} = \frac{d(S+E+I+R)}{dt} = \Lambda - \delta(S+E+I+R) - dI,$$

$$\Rightarrow \quad i.e. \qquad \frac{dP}{dt} + \delta P \leq \Lambda$$

Integrating both sides of the above inequality, we get

$$0 < P(t) \leq \frac{\Lambda}{\delta} + P(0)e^{-\delta t},$$

where, $P(0) = S(0) + E(0) + I(0) + R(0)$. Thus, we have $P(t) = \frac{\Lambda}{\delta}$ as $t \to \infty$. Therefore, all the solution of the system (1) that initiating in $\{\mathbb{R}_+^4\}$ are confined in the region

$$\Gamma = \left\{(S+E+I+R) \in \mathbb{R}_+^4 : 0 < S+E+I+R \leq \frac{\Lambda}{\delta} + \tau\right\}$$

for any $\tau > 0$ and for $t \to \infty$. The region Γ is positively invariant and attracting region. The well-posedness of the system is established in this way. Hence the proof.

Table 1: Relevant parameters values used for numerical simulations of the system (1).

Parameters	Assigned Value
Λ	10
ρ	0.7
β	0.05
ϵ	[0.1 – 1.25]
γ	1.2
η	0.4
d	0.2
ξ	1.25
δ	[0.05, 0.2]

4 Equilibrium points of the system

The $SEIR$ epidemic system (1) executes two steady states namely –

(i) the disease-free equilibrium (DFE) $E_0 = (\Lambda/\delta, 0,0,0)$, which always exists and

(ii) the endemic equilibrium (EE) $E^* = (S^*, E^*, I^*, R^*)$, whose existence conditions would be studied.

The components of the endemic equilibrium E^* are computed as

$$S^* = \frac{(\gamma+\delta)(\eta+d+\xi+\delta) + \Lambda\epsilon\gamma}{\gamma\big((1-\rho)\beta+\delta\epsilon\big)}, E^* = \frac{\delta(\eta+d+\xi+\delta)(R_0-1)}{\gamma\big((1-\rho)\beta+\delta\epsilon\big)},$$

$$I^* = \frac{\delta(R_0-1)}{(1-\rho)\beta+\delta\epsilon}, R^* = \frac{(\eta+\xi)(R_0-1)}{(1-\rho)\beta+\delta\epsilon}.$$

It is obtained that EE exists when $R_0 > 1$.

4.1 Basic reproduction number

Using the Next-Generation Matrix (NGM) method, we compute the basic reproduction number of the system [13, 14]. Basic reproduction number plays the center role in analyzing dynamics of any communicable disease. Let us assume that F be the emergence rate of new infections and V be the transition rate of infection between the infected compartments E and I at the DFE E_0 which are defined as follows:

$$F = \begin{pmatrix} (1-\rho)\frac{\beta SI}{1+\epsilon I} \\ 0 \end{pmatrix} and V = \begin{pmatrix} (\gamma+\delta)E \\ -\gamma E + (\eta+d+\xi+\delta)I \end{pmatrix}.$$

Thus according to [13] , we get the entrywise non-negative matrices F and V as

$$F = \begin{pmatrix} 0 & (1-\rho)\frac{\beta\Lambda}{\delta} \\ 0 & 0 \end{pmatrix} and\ V = \begin{pmatrix} (\gamma+\delta) & 0 \\ -\gamma & (\eta+d+\xi+\delta) \end{pmatrix}.$$

Henceforth, the next-generation matrix defining the expected value of secondary infections is defined as

$$FV^{-1} = \begin{pmatrix} \frac{(1-\rho)\beta\Lambda\gamma}{\delta(\gamma+\delta)(\eta+d+\xi+\delta)} & \frac{(1-\rho)\beta\Lambda}{\delta(\eta+d+\xi+\delta)} \\ 0 & 0 \end{pmatrix}.$$

The basic reproduction number, R_0, the spectral radius (r) of FV^{-1} be designated as

$$r(FV^{-1}) = R_0 = \frac{(1-\rho)\beta\Lambda\gamma}{\delta(\gamma+\delta)(\eta+d+\xi+\delta)}.$$

With the help of this basic reproduction number (R_0), a threshold, we will perform theoretical analysis of the system (1).

5 Local dynamics of the system

Theorem 3. *The SEIR system (1) is LAS (locally asymptotic stable) around the DFE E_0 whenever $R_0 < 1$; otherwise, the system would be unstable for $R_0 > 1$.*

Proof. To analyze the local stability of the *SEIR* epidemic system (1) around the DFE E_0, first we have to compute the Jacobian matrix with reference to the system (1) at the DFE which is defined as follows:

$$J_0 = \begin{pmatrix} -\delta & 0 & 0 & 0 \\ 0 & -(\gamma+\delta) & \frac{(1-\rho)\beta\Lambda}{\delta} & 0 \\ 0 & \gamma & -(\eta+d+\xi+\delta) & 0 \\ 0 & 0 & (\eta+\xi) & -\delta \end{pmatrix}.$$

From the characteristic equation for the Jacobian J_0 corresponding to the eigenvalue λ it can be observed that two eigen values are negative $(-\delta, -\delta)$ and the rest two eigenvalues can be determined from the following quadratic equation

$$\lambda^2 + a_1\lambda + a_0 = 0, \tag{7}$$

where

$$\begin{aligned} a_1 &= (\gamma+\eta+d+\xi+2\delta), \\ a_0 &= (\gamma+\delta)(\eta+d+\xi+\delta) - \frac{(1-\rho)\beta\Lambda\gamma}{\delta}. \end{aligned}$$

The equation (7) reveals that for the stability of the DFE, the rest two eigenvalues should be negative or have negative real parts and that would be possible only when $a_0 > 0$ that is $R_0 < 1$. Therefore, the disease-free equilibrium (E_0) would be LAS (locally asymptomatic stable) and the elimination of infection would take place from the system if and only if $R_0 < 1$ and the DFE would be unstable if R_0 would be greater than unity.

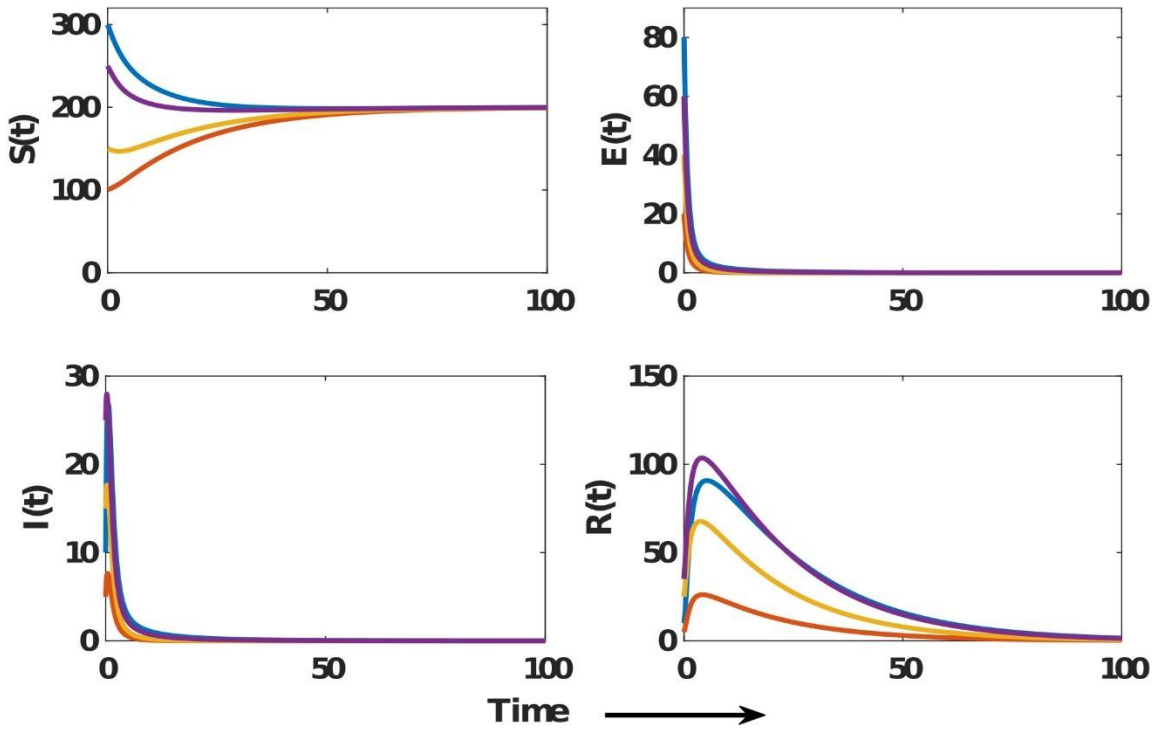

Figure 2: The figure is portraying solution trajectories of the system (1) for different initial conditions. The baseline parameter values are same as they are enlisted in Table 1 whenever $R_0 = 0.9954 < 1$.

Theorem 4. *The SEIR system (1) is LAS (locally asymptotic stable) around the EE E^* whenever $R_0 > 1$; otherwise, the system would be unstable for $R_0 < 1$.*

Proof. To analyze the local stability of the *SEIR* system (1) around the EE $E^* = (S^*, E^*, I^*, R^*)$, first we have to compute the Jacobian matrix with reference to the system (1) at the EE which is defined as follows:

$$J^* = \begin{pmatrix} -(1-\rho)\dfrac{\beta I^*}{1+\epsilon I^*} - \delta & 0 & -(1-\rho)\dfrac{\beta S^*}{(1+\epsilon I^*)^2} & 0 \\ (1-\rho)\dfrac{\beta I^*}{1+\epsilon I^*} & -(\gamma+\delta) & (1-\rho)\dfrac{\beta S^*}{(1+\epsilon I^*)^2} & 0 \\ 0 & \gamma & -(\eta+d+\xi+\delta) & 0 \\ 0 & 0 & (\eta+\xi) & -\delta \end{pmatrix}.$$

The characteristic equation for the Jacobian J^* corresponding to the eigenvalue λ_1 is given by

$$a_3\lambda_1^3 + a_2\lambda_1^2 + a_1\lambda_1 + a_0 = 0, \tag{8}$$

where,

$$\begin{aligned} a_3 &= 1, \\ a_2 &= (\eta+d+\xi+3\delta+\gamma) + (1-\rho)\frac{\beta I^*}{1+\epsilon I^*}, \\ a_1 &= (\gamma+\delta)(\eta+d+\xi+\delta) + \delta(\eta+d+\xi+2\delta+\gamma) + (1-\rho)\frac{\beta I^*}{1+\epsilon I^*}\left[1 - \frac{\gamma}{(1+\epsilon I^*)}\right], \\ a_0 &= \delta(\gamma+\delta)(\eta+d+\xi+\delta) + (1-\rho)\frac{\beta}{1+\epsilon I^*}\left[I^*(\gamma+\delta)(\eta+d+\xi+\delta) - \gamma\delta\frac{S^*}{1+\epsilon I^*}\right] \end{aligned}$$

To analyze the stability of the epidemic system (1) around the EE E^*, we take help of well-known Routh-Hurwitz criteria of stability From the characteristic equation (8), it could be seen that one eigenvalue is δ which is strictly negative and purely real. To check the Routh-Hurwitz criteria is satisfied not, we have to check whether $(i) a_i^S > 0$, for $i = 0,1,2,3$ and $a_2a_1 > a_3a_0$. It could be observed that Routh-Hurwitz criteria would be satisfied only if $(i) I^* > \frac{\gamma-1}{\epsilon}$ and $(ii) R_0 > 1$. Thus, the epidemic system (1) is LAS around the EE E^* only if $(i) I^* > \frac{\gamma-1}{\epsilon}$ and $(ii) R_0 > 1$. Otherwise, system would be unstable. Hence the proof. □

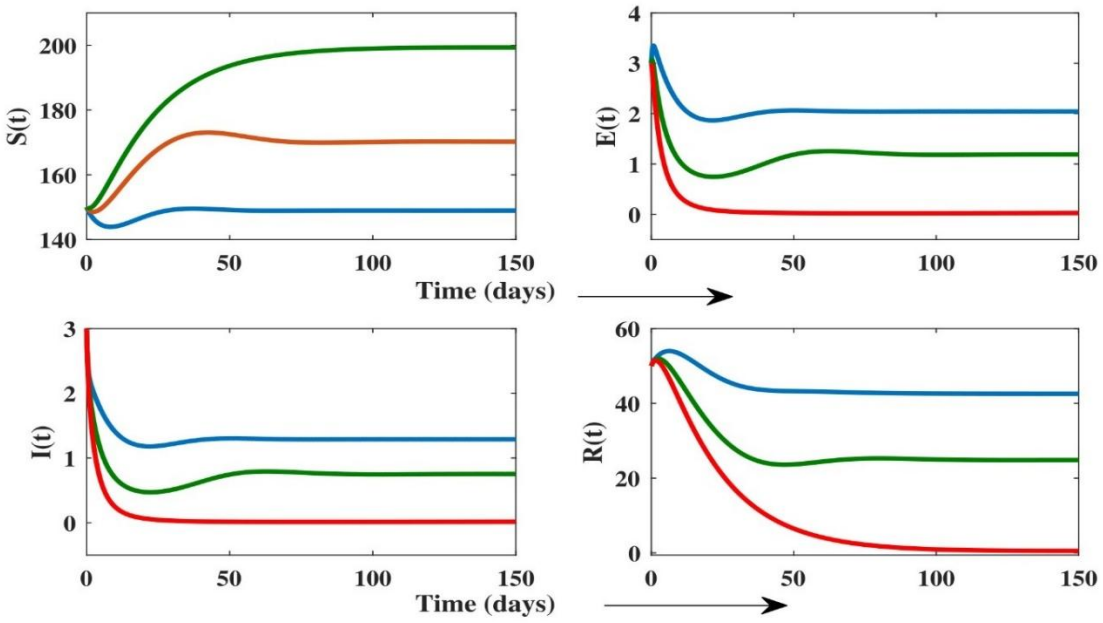

Figure 3: The figure is portraying the solution trajectories of the epidemic system (1) for $R_0 > 1$ and varying the rate of accurate media awareness, $\rho = 0.7,0.75,0.8$. Rest parameter values are same as they are enlisted in Table 1.

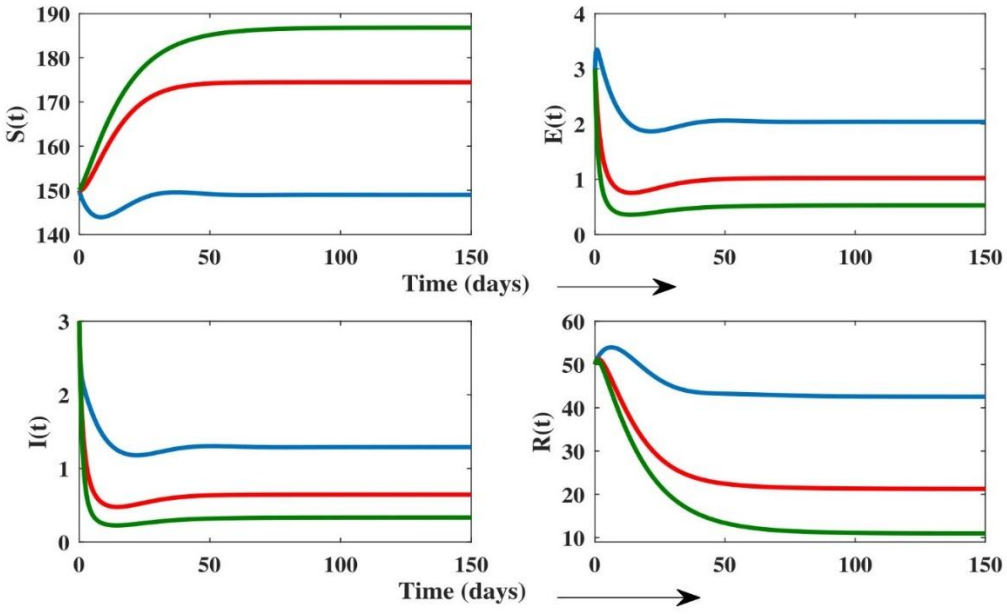

Figure 4: The figure is portraying the solution trajectories of the epidemic system (1) for $R_0 > 1$ and varying the rate of psychological fear, ⊚ = *0.1,0.5,1.25*. Rest parameter values are same as they are enlisted in Table 1.

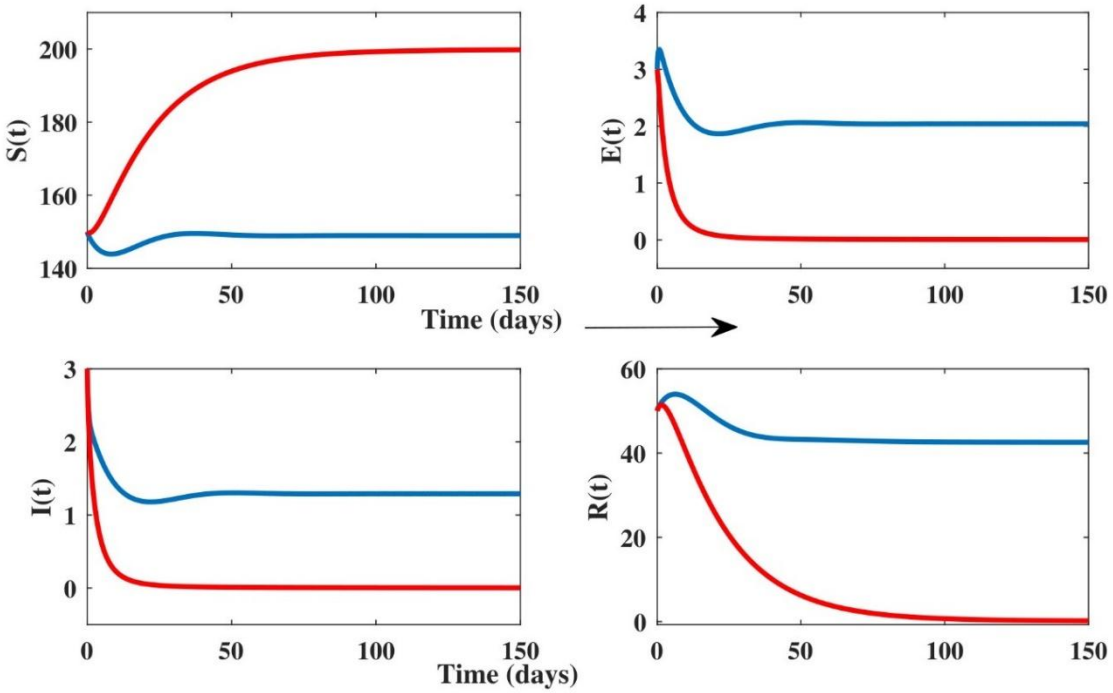

Figure 5: The figure is portraying the solution trajectories of the epidemic system (1) for $\rho = 0.8$, $R_0 > 1$, and $\rho = 0.803$, $R_0 < 1$. Rest parameter values are same as they are enlisted in Table 1.

6 Global dynamics of the system

Theorem 5. *The SEIR epidemic system (1) is GAS (globally asymptotically stable) around the DFE E_0 whenever $R_0 < 1$; otherwise, the system would be unstable for $R_0 > 1$.*

Proof. It is obtained from the $SEIR$ epidemic system (1) that S, and R are the disease-free classes of the epidemic system (1) and the infected classes are E, and I. Thus, we can rearrange the $SEIR$ epidemic system equations (1) as

$$\frac{dX}{dt} = P(X,Y)$$
$$\frac{dY}{dt} = G(X,Y), G(X,0) = 0,$$

where the compartments could be written as

$$X = (S,R) \in \mathbb{R}^2_+,$$
$$Y = (E,I) \in \mathbb{R}^2_+.$$

Now, following the approach implemented by Castillo-Chavez [15, 16] , we are aimed to study the conditions of global stability of the epidemic system (1) around the disease-free equilibrium point $E_0 = (\Lambda/\delta, 0,0,0)$. To analyze the global stability of $SEIR$ epidemic system (1), the system equations (1) must satisfy the following two conditions:

1. $\frac{dX}{dt} = P(X,0)$, X^* is globally asymptotic stable.

2. $G(X,Y) = KY - G(X,Y), G(X,Y) \geq 0$ where $K = D_Y G(X^*,0)$ is the Metzler Matrix and $(X,Y) \in \Gamma$. The region Γ is biological feasible and attracting where all solutions of the system (1) initiating from $\mathbb{R}^4_+$ will enter into the interior of the region Γ and will never decamp from the region. Consequently, from the system (1), it could be computed that

$$P(X,0) = \begin{pmatrix} \Lambda - \delta S \\ 0 \end{pmatrix},$$

$$K = \begin{pmatrix} -(\gamma+\delta) & 0 \\ \gamma & -(\eta+d+\xi+\delta) \end{pmatrix},$$

$$and\ G = \begin{pmatrix} (1-\rho)\dfrac{\beta SI}{1+\epsilon I} \\ 0 \end{pmatrix}.$$

Consequently, it is observed that for $(X,Y) \in \Gamma$, $G(X,Y) \geq 0$ and it could be found that $X^* = (\Lambda/\delta, 0)$ is globally asymptotically stable equilibrium point of the limiting system, $\frac{dX}{dt} = P(X,0)$. Hence, the two above stated conditions are satisfied for $R_0 < 1$. Thus, the $SEIR$ epidemic system (1) is globally asymptotically stable around the disease-free equilibrium point $E_0 = (\Lambda/\delta, 0,0,0)$ is while $R_0 < 1$. Otherwise, it would be unstable. □

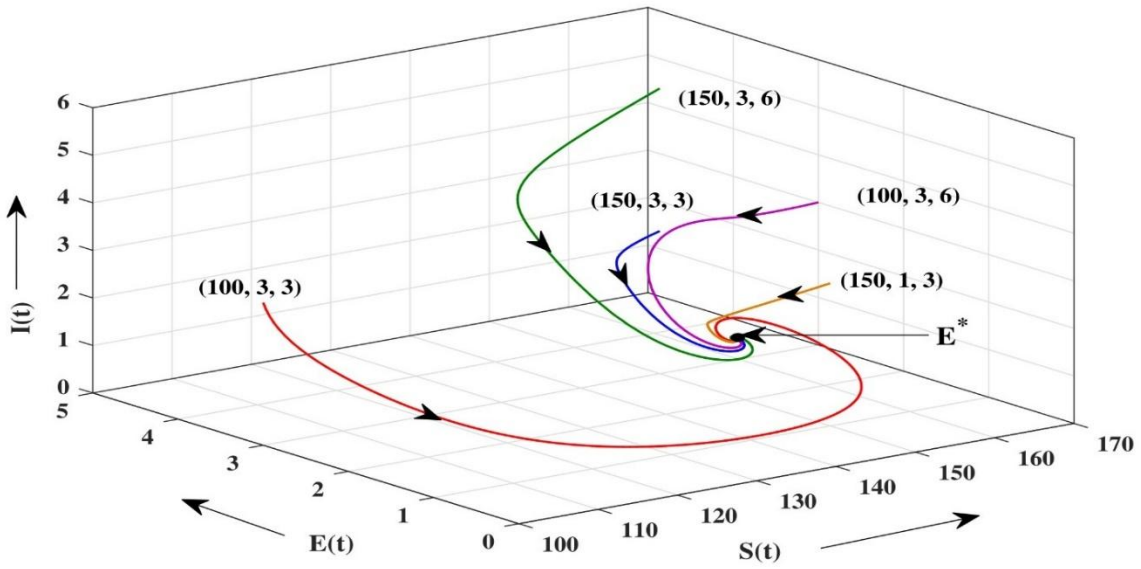

Figure 6: The figure is portraying the global stability of the epidemic system (1) around the endemic equilibrium point irrespective of initial conditions in the $S-E-I$ phase space with $\rho = 0.7$ and $R_0 = 1.516 > 1$.

7 Sensitivity analysis of the endemic equilibrium

In this section, the sensitivity of the endemic equilibrium $E^* = (S^*, E^*, I^*, R^*)$ of the $SEIR$ epidemic system (1) would be analyzed following the technique of Chitnis et al.[19] . Sensitivity analysis is used in order to determine the influence and robustness of model parameters value related to the endemic equilibrium $E^* = (S^*, E^*, I^*, R^*)$ in disease prevalence, disease progression and overall disease dynamics.

Parameters associated to E^* which have sensitivity indices greater than zero, imply the fact that increasing of this parametric value, would increase the values of components associated to E^*. Similarly, for the parameters associated to E^* having sensitivity indices less than zero, imply the fact that increasing of this parametric value, would decrease the the values of components associated to E^*. The sensitivity indices of the parameters related to components of the endemic equilibrium are computed and enlisted in the following Table 2.

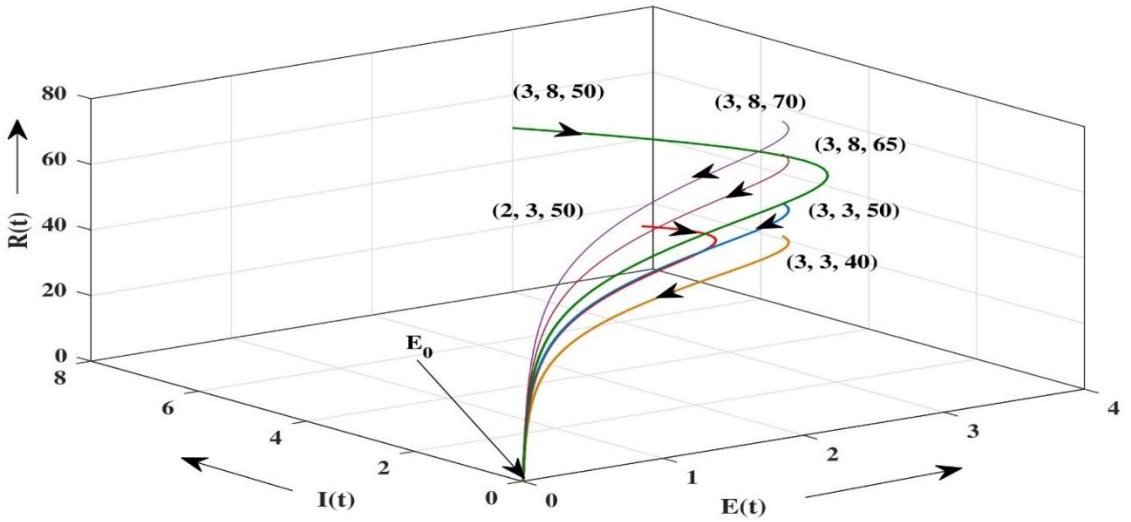

Figure 7: The figure is portraying the global stability of the epidemic system (1) around the disease-free equilibrium point irrespective of initial conditions in the $E - I - R$ phase space with $\rho = 0.803$ and $R_0 = 0.9954 < 1$.

From the Table 2, it is obtained that the parameters which are positively correlated with S^* are Λ, ρ, ϵ, γ, η, ξ, and d and the parameters which are negatively correlated with S^* are β and δ. From the Table 2, it is obtained that the parameters which are positively correlated with E^* are Λ, β, γ and d and the parameters which are negatively correlated with E^* are ρ, ϵ, η, ξ and δ. From the Table 2, it is obtained that the parameters which are positively correlated with I^* are Λ, ρ, β and γ. Again, the parameters which are negatively correlated with I^* are ϵ, η, ξ, d and δ. From the Table 2, it is obtained that the parameters which are positively correlated with R^0 are Λ, ρ, β and γ. Again, the parameters which are negatively correlated with I^* are ϵ, η, ξ, d and δ. Increasing the value of parameters that are positively correlated to E^*, the level of infection would be increased. Henceforth, the value of these parameters must be reduced.

Table 2: Sensitivity indices of the parameters related to E^* of the $SEIR$ system (1).

Parameters	Sensitivity Indices			
	S^*	E^*	I^*	R^*
Λ	+0.863309353	+2.448979592	+0.758437019	+1.053883451
ρ	+0.451612903	-5.337941628	+2.175115207	+0.780024848
β	-0.193548387	+2.287689269	+2.745227123	+0.822520148
ϵ	+0.175214667	-0.672043011	-0.806451613	-0.289204128
γ	+0.227625899	+0.354010025	+1.082706767	+0.175647242
η	+0.043741007	-0.517006803	-0.508734694	-0.250306191
ξ	+0.136690647	-1.615646259	-1.589795918	-0.782206846
d	+0.021870504	+0.138980323	-0.254367347	-0.168621352
δ	-0.061797482	-2.098425788	-2.637744361	-1.123772564

8 Sensitivity analysis of basic reproduction number

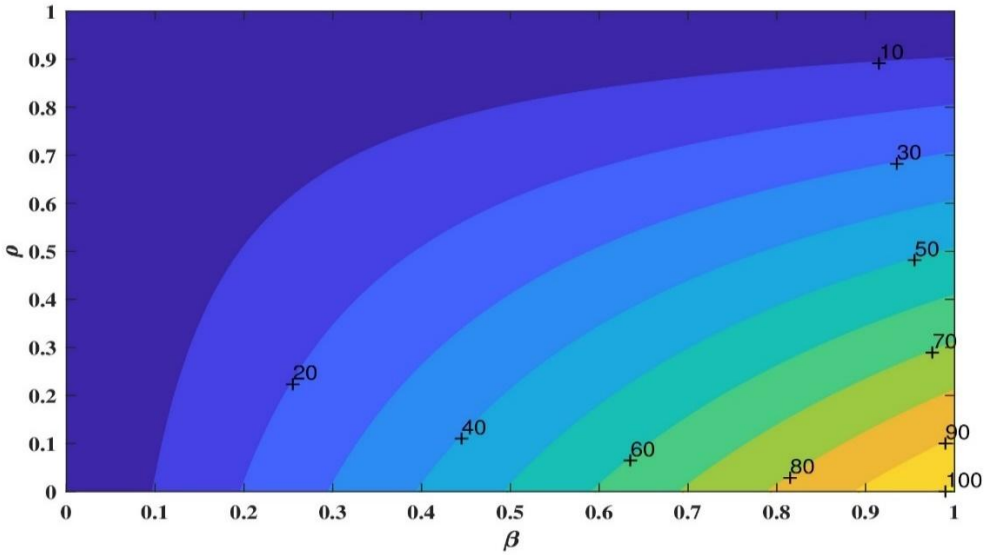

Figure 8: The figure is portraying contour plot of sensitivity of R_0 varying the rate of disease-transmission (β) and the rate of accurate media awareness (ρ).

In this section, we would carry out normalized forward sensitivity indices of the baseline parameter values associated to the basic reproduction number, R_0 . The normalized forward sensitivity index of R_0 with respect to the rate of disease transmission (β) is given by

$$\Upsilon_{\beta}^{R_0} = \frac{\partial R_0}{\partial \beta} \times \frac{\beta}{R_0}.$$

Table 3: Sensitive indices of the model parameters associated to R_0 of $SEIR$ model (1)

Parameters	Sensitivity indices
Λ	+1
β	+1
ρ	-0.23330
γ	+0.03990
η	-0.21053
d	-0.21053
ξ	-0.21053
δ	-1

Using the baseline parameter values, it is obtained that $\Upsilon_{\beta}^{R_0} = +1$. This computed sensitivity index $\Upsilon_{\beta}^{R_0}$ implies that for increasing the parameter β by 10% will increase R_0 by 10%. Therefore, it could be concluded that in order to control any infection, the transmission rate (β) must be reduced. These normalized forward sensitivity indices are used used to measure the relative influences of the baseline parameter values on the overall disease dynamics process. The knowledge about influences of baseline parameters aids in determining proper intervention strategies.

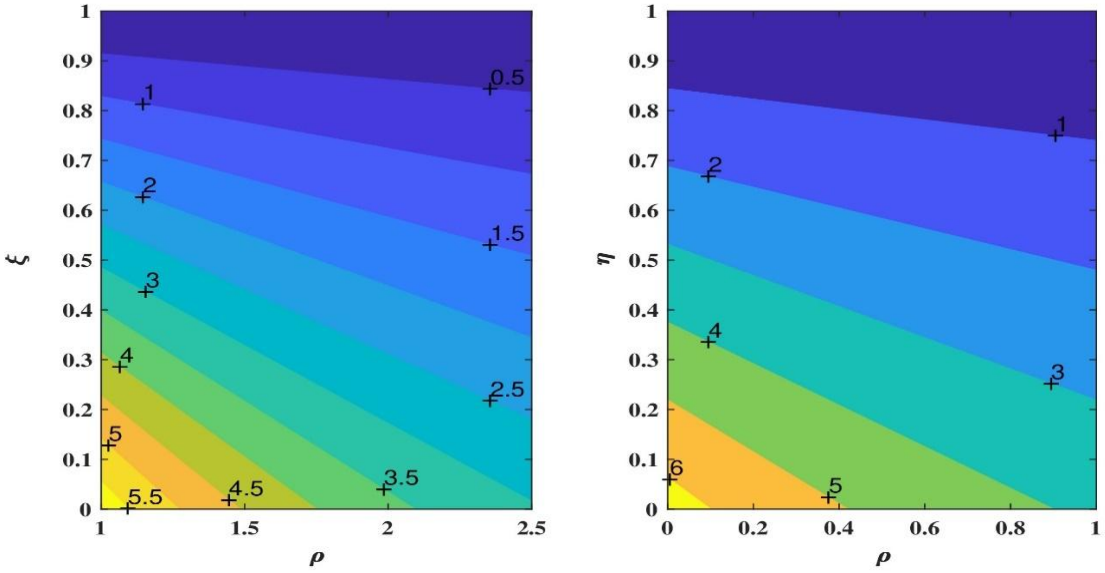

Figure 9: The figure is portraying contour plot of sensitivity of R_0 varying the rate of recovery through pharmaceutical interventions (ξ) against the rate of accurate media awareness (ρ) (in left panel) and the rate of natural recovery through immunity (η) against the rate of accurate media awareness (ρ) (in right panel).

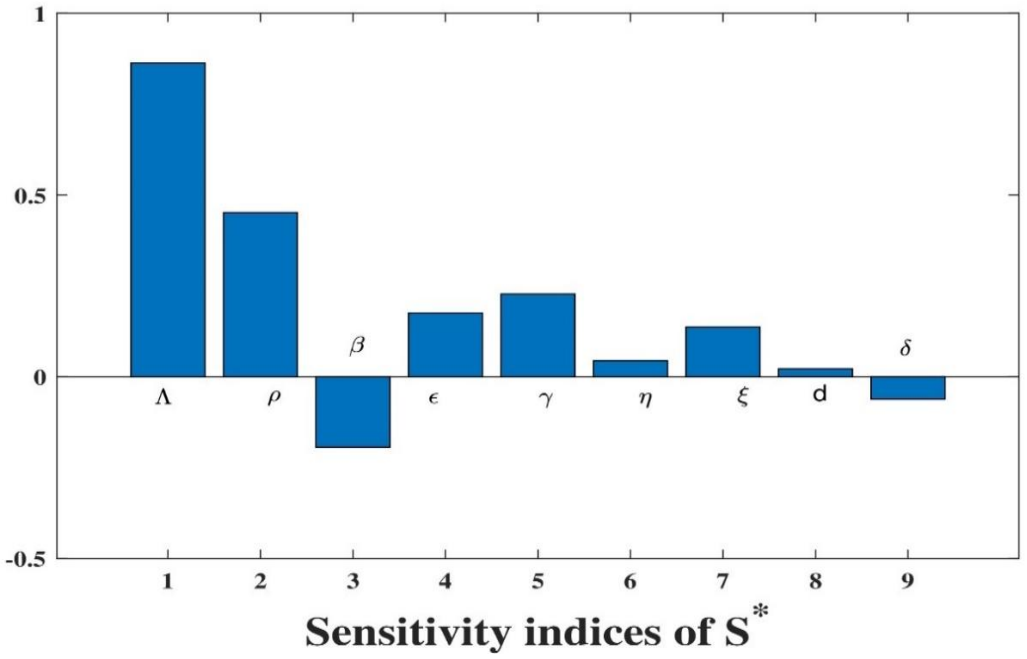

Figure 10: The figure is portraying tornado plot of sensitivity of the parameters related to S^* component of the endemic equilibrium point, E^*.

Now, we enlist the normalized forward sensitivity indices of the parameters associated to the basic reproduction number R_0 in Table 3 and it would be noticed that some parameters have positive sensitivity indices and some have negative sensitivity indices. Positive indices imply that, increase the value of these parameters will increase the value of R_0 and negative indices imply that, increase the value of these parameters will decrease the value of R_0. We should pay more attention to the most influential parameters. The normalized forward sensitivity indices indicate that the most sensitive parameters are constant recruitment of susceptible individuals (Λ), effective rate of disease transmission (β), accurate media awareness (ρ) and rate of natural morbidity (δ). Thus, it is obtained that constant recruitment of susceptible individuals and effective rate of disease transmission must be reduced in controlling transmission. Again, accurate media awareness must be increased to control psychological fear and to curtail the chain of disease progression.

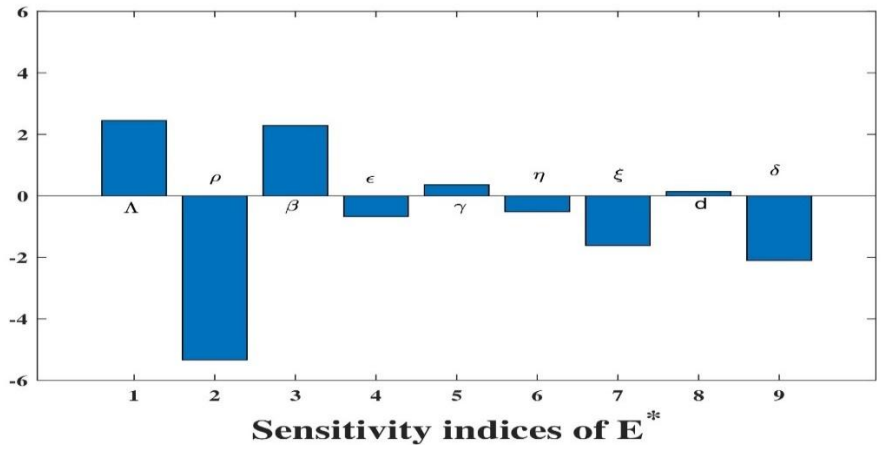

Figure 11: The figure is portraying tornado plot of sensitivity of the parameters related to E^* component of the endemic equilibrium point, E^*.

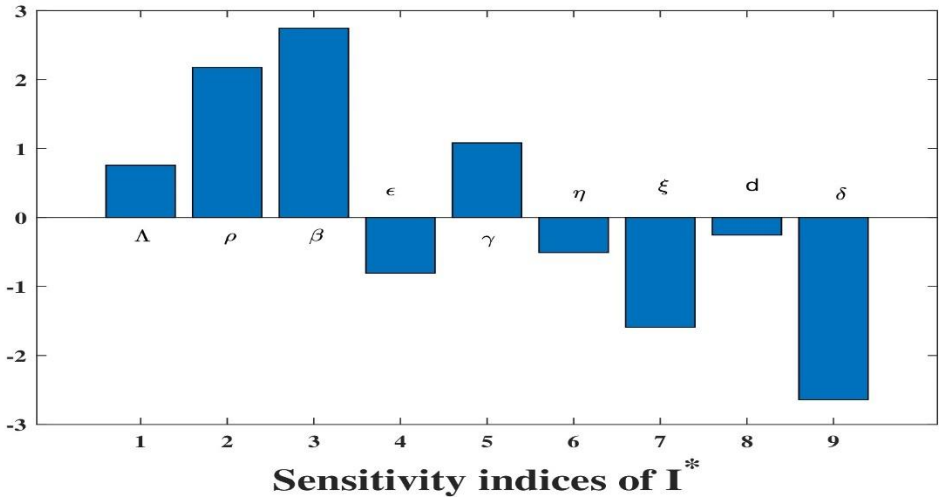

Figure 12: The figure is portraying tornado plot of sensitivity of the parameters related to I^* component of the endemic equilibrium point, E^*.

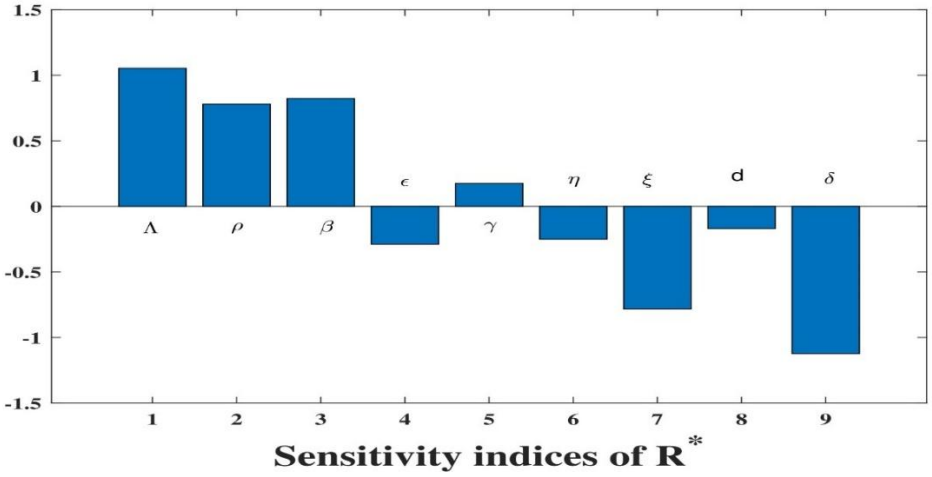

Figure 13: The figure is portraying tornado plot of sensitivity of the parameters related to R^* component of the endemic equilibrium point, E^*.

9 Numerical Simulation

In this section, we are aimed to simulate our proposed $SEIR$ epidemic system (1) numerically with the help of baseline parameter values enlisted in Table 1 and MATLAB software. For the baseline parameter values enlisted in Table 1, it is observed that the epidemic system (1) possesses two equilibrium points - (i) one disease-free equilibrium point (200, 0, 0, 0) and (ii) one endemic equilibrium point (148.958, 2.04167, 1.28947, 42.5526). Also, it is obtained that the basic reproduction number of the system, $R_0 = 1.516 > 1$, for the accurate media awareness rate (ρ) = 0.7 and psychological fear (ϵ) = 0.1. Again, we take the accurate media awareness rate (ρ) = 0.803 and then $R_0 = 0.9954 < 1$. In Figure 2, time series solution of the $SEIR$ epidemic system (1) is calibrated taking ρ = 0.803 and $R_0 = 0.9954 < 1$. Figure 3 is indicating the time series evolution of the $SEIR$ epidemic system (1) for different value of ρ (ρ = 0.7, 0.75, 0.8). The figure is showing that for increasing value of awareness level of infection would be lowered and rate of recovery would be improved. In Figure 4, time series evolution of the epidemic system (1) is portrayed varying the rate of psychological fear, $\epsilon = 0.1, 0.5, 1.25$. From Figure 4, it is obtained that a certain level of self-imposed psychological fear is necessary to cut the disease-progression chain but it would hamper the immunology of a patient so that recovery process would be delayed. Figure 5 is portraying time series solution for two different values of accurate media awareness rate (ρ), $\rho = 0.7$ and $\rho = 0.803$. Figure 6 is portraying the global stability of the epidemic system (1) around the

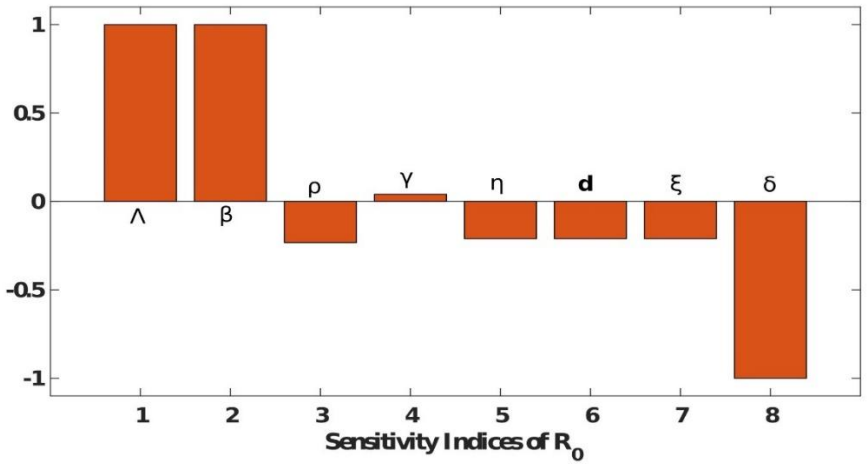

Figure 14: The figure is portraying tornado plot of sensitivity of the parameters related to the basic reproduction number, R_0.

endemic equilibrium point E^* irrespective of initial conditions in the $S - E - I$ phase space with $\rho = 0.7$ and $R_0 = 1.516 > 1$. Figure 7 is portraying the global stability of the epidemic system (1) around the disease-free equilibrium point E_0 irrespective of initial conditions in the $E - I - R$ phase space with $\rho = 0.803$ and $R_0 = 0.9954 < 1$.

In Figure 8, contour plot of sensitivity of the baseline parameters associated to the basic reproduction number (R_0) varying the rate of disease-transmission (β) and the rate of accurate media awareness (ρ) is portrayed. The figure is indicating that rate of disease-transmission should be decreased and the rate of accurate media awareness should be increased to reduce the value of R_0 below unity. In Figure 9, contour plot of sensitivity of R_0 varying the rate of recovery through pharmaceutical interventions (ξ) against the rate of accurate media awareness (ρ) (in left panel) and the rate of natural recovery through immunity (η) against the rate of accurate media awareness (ρ) (in right panel) are portrayed. The figure is indicating that simultaneous increment in accurate media awareness and applications pharmaceutical interventions along with healthy immunity are able to reduce the value of R_0 below unity.
Figure 10, Figure 11, Figure 12 and Figure 13 are representing the tornado plots of sensitivity of the baseline parameters associated to the S^*, E^*, I^*, and R^* components of the endemic equilibrium point, $E^* = (S^*, E^*, I^*, R^*)$. Figure 14 is representing the tornado plot of sensitivity of the baseline parameters associated to the basic reproduction number, R_0.

10 Conclusions

Calibrating the influences of self-imposed psychological fear and appropriate media awareness in transmission dynamics of an epidemic, a classic $SEIR$ compartmental, deterministic ODE model is proposed and analyzed comprehensively. The epidemic system possesses two equilibrium points - one disease-free equilibrium point and another endemic equilibrium point. The local stability criteria of the epidemic system around both the equilibrium points are studied. Furthermore, global stability of the

proposed system around the disease-free equilibrium point is investigated. Thereafter, the sensitivity of the model parameters associated to the endemic equilibrium point is analyzed. We computed all the sensitivity indices of the model parameters related to all the four components of endemic equilibrium point indicating the influences of the model parameters on disease transmission process. Also, we compute the normalized forward sensitivity indices of the baseline parameters associated to the basic reproduction number. It is obtained that the most influential parameters are constant recruitment of susceptible individuals (Λ), effective rate of disease transmission (β), accurate media awareness (ρ) and rate of natural morbidity (δ).

The impact of self-imposed psychological fear is very significant factor measuring the degree of individual's response during onset of an epidemic and available intervention strategies. To some extent, this psychological fear is necessary to decrease the level of infection. Accompanying this positive impact of fear, accurate media awareness is essential to aware individuals about the traits of an infection, non-pharmaceutical measures, and available pharmaceutical measures and to mitigate the global burden of deadly infectious disease. Heath policy makers should pay more attention to aware human beings in a large scale through accurate mass media presentations.

Data accessibility. All data generated or analyzed during this study are included in this article.

Authors' contributions. **Piu Samui:** Formal analysis, Conceptualization, Software, Validation, Methodology, Resources, Writing – original draft, Writing – review & editing, Visualization, Supervision.

Conflict of interest declaration. The author declares that there is no conflict of interests regarding the publication of this article.

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References

1. Ghosh, I., Tiwari, P. K., Samanta, S., Elmojtaba, I. M., Al-Salti, N., & Chattopadhyay, J. (2018). A simple SI-type model for HIV/AIDS with media and self-imposed psychological fear. Mathematical Biosciences, 306, 160-169.
2. Mondal, J., Samui, P., & Chatterjee, A. N. (2021). Optimal control strategies of non-pharmaceutical and pharmaceutical interventions for COVID-19 control. Journal of Interdisciplinary Mathematics, 24(1), 125-153.
3. Thirthar, A. A., Abboubakar, H., Khan, A., & Abdeljawad, T. (2023). Mathematical modeling of the COVID-19 epidemic with fear impact. AIMS Math, 8(3), 6447-6465.
4. Samui, P., Mondal, J., & Khajanchi, S. (2020). A mathematical model for COVID-19 transmission dynamics with a case study of India. Chaos, Solitons & Fractals, 140, 110173.
5. Misra, A. K., Sharma, A., & Shukla, J. B. (2011). Modeling and analysis of effects of awareness programs by media on the spread of infectious diseases. Mathematical and Computer Modelling, 53(5-6), 1221-1228.
6.Mondal, J., Khajanchi, S., & Samui, P. (2022). Impact of media awareness in mitigating the spread of an infectious disease with application to optimal control. The European Physical Journal Plus, 137(8), 983.
7.Misra, A. K., Rai, R. K., & Takeuchi, Y. (2018). Modeling the control of infectious diseases: Effects of TV and social media advertisements. Mathematical Biosciences & Engineering, 15(6), 1315-1343.
8. Bozkurt, F., Yousef, A., Abdeljawad, T., Kalinli, A., & Al Mdallal, Q. (2021). A fractional-order model of COVID-19 considering the fear effect of the media and social networks on the community. Chaos, Solitons & Fractals, 152, 111403.
9. Misra, A. K., Tripathi, K. K., & Verma, M. (2024). Modeling the effects of psychological fear and media-induced awareness on the dynamics of infectious diseases. Discrete and Continuous Dynamical Systems - Series B, 29(2), 909-936.
10. Samanta, S., Rana, S., Sharma, A., Misra, A. K., & Chattopadhyay, J. (2013). Effect of awareness programs by media on the epidemic outbreaks: A mathematical model. Applied Mathematics and Computation, 219(12), 6965-6977.
11. Shanta, S. S., & Biswas, M. H. A. (2020). The Impact of Media Awareness in Controlling the Spread of Infectious Diseases in Terms of SIR Model. Mathematical Modelling of Engineering Problems, 7(3).
12. Yousef, A., Bozkurt, F., Abdeljawad, T., & Emreizeeq, E. (2023). A mathematical model of COVID-19 and the multi fears of the community during the epidemiological stage. Journal of Computational and Applied Mathematics, 419, 114624.
13. Van den Driessche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Mathematical Biosciences, 180(1-2), 29-48.
14. Mondal, J., Samui, P., Chatterjee, A. N., & Ahmad, B. (2024). Modeling hepatocyte apoptosis in chronic HCV infection with impulsive drug control. Applied Mathematical Modelling, 136, 115625.

15. Castillo-Chavez, C., Blower, S., van den Driessche, P., Kirschner, D., & Yakubu, A. A. (Eds.). (2002). Mathematical approaches for emerging and reemerging infectious diseases: models, methods, and theory (Vol. 126). Springer Science & Business Media.
16. Mondal, J., & Khajanchi, S. (2022). Mathematical modeling and optimal intervention strategies of the COVID-19 outbreak. Nonlinear Dynamics, 109(1), 177-202.
17. Mondal, J., Samui, P., & Chatterjee, A. N. (2022). Modelling of contact tracing in determining critical community size for infectious diseases. Chaos, Solitons & Fractals, 159, 112141.
18. Mondal, J., Samui, P., & Chatterjee, A. N. (2021). Effect of SOF/VEL antiviral therapy for HCV treatment: a control theoretic approach. Letters in Biomathematics, 8(1), 191-213.
19. Chitnis, N., Hyman, J. M., & Cushing, J. M. (2008). Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. Bulletin of Mathematical Biology, 70, 1272-1296.